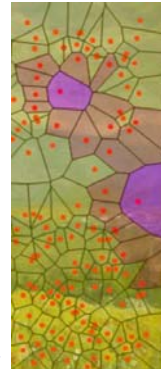


# Zdzisław Pawlak & Claude Monet Painting Maximal Nucleus Clusters

James F. Peters and Sheela Ramanna

Computational Intelligence Laboratory, University of Manitoba  
[james.peters3@umanitoba.ca](mailto:james.peters3@umanitoba.ca)



**Setting** : Places painted by Prof. Zdzisław Pawlak and Claude Monet(CM).

**Method** : keypoint-based Voronoï Meshes, maximal nucleus clusters (MNCs), Rényi entropy of the MNCs.

**Landscapes** : Comparison of Voronoï Meshes on Paintings.

**Nature Trails** : Comparison of Voronoï Meshes on Paintings.

**Wild Flowers** : Comparison of Voronoï Meshes on Paintings.

**Conjectures** : About Paintings by Pawlak & Monet.

**Application** : Comments on Classification of Paintings.

# Setting for Paintings by Monet and Pawlak

**Prezelom Dunajca** : Dunajec river gorge running through the Pieniny mountains in southern Poland.

**Kampinoski Park Narodowy** :  
National Park, northwest outskirts of Warszawa [Warsaw].

Favorite places for paintings by Prof. Zdzisław Pawlak .

**Vétheuil** : arrondissement of Pontoise in the Val-d'Oise department, northwestern suburb of Paris, France. Impressionist painter Claude Monet lived in Vétheuil, 1878-1881.

Favorite place for paintings by Claude Monet .

# Method in Analysing Paintings by Monet and Pawlak

**SIFT keypoints** : set  $S =$  image pixels with different gradient orientations and gradient magnitudes.

**Voronoi Region** :

$$V(s) = \{x \in \mathbb{R}^2 : \|x - s\| \leq \|x - q\|, \text{ for all } q \in S\}.$$

**Voronoi Mesh** :

$$V(S) = \bigcup_{s \in S} V(s) \text{ (Voronoi mesh).}$$

**Maximal Nucleus Cluster** :

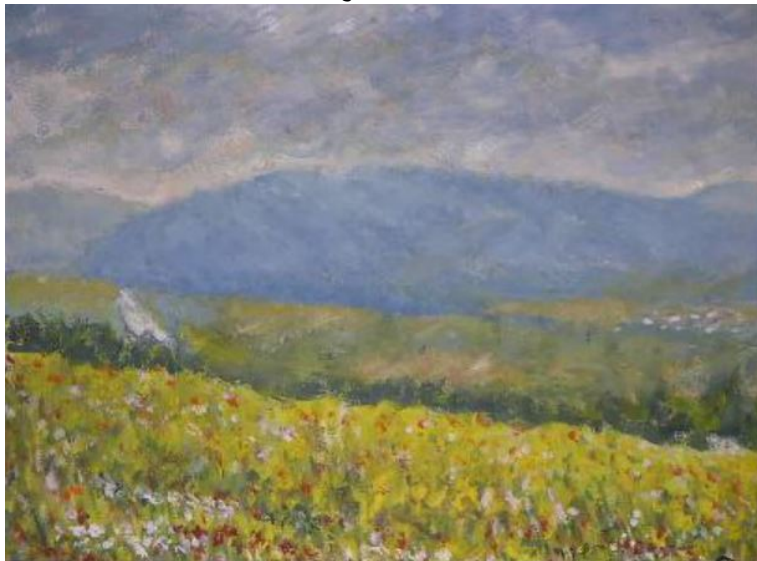
$$N \in V(S),$$

$$\mathfrak{C} N = \left\{ A \in V(S) : \text{cl } A \stackrel{\mathfrak{M}}{\delta} N \right\} \text{ (Nucleus cluster),}$$

$$MNC(V(S)) = \max_{N \in V(S)} \{\mathfrak{C} N\}.$$

# Pawlak (Prezelom Dunajca) & Monet (Vetheuil)

## Pawlak Prezelom Dunajca Vista



Pawlak Prezelom Dunajca Vista



# Pawlak & Monet Painting Maximal Nucleus Clusters

2016-09-08

└ Pawlak (Prezelom Dunajca) & Monet (Vetheuil)



CM

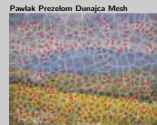
## Pawlak Prezelom Dunajca Mesh



2016-09-08

# Pawlak & Monet Painting Maximal Nucleus Clusters

└ Pawlak (Dunajca Mesh) & Monet (Vetheuil Mesh)



CM



## Pawlak Prezelom Dunajca Nuclei



2016-09-08

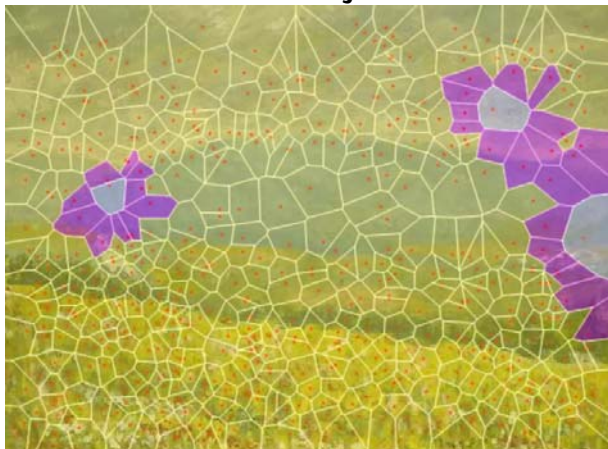
# Pawlak & Monet Painting Maximal Nucleus Clusters

└ Pawlak (Dunajca Nuclei) & Monet (Vetheuil Nucleus)



CM

## Pawlak Prezelom Dunajca MNCs



2016-09-08

## Pawlak & Monet Painting Maximal Nucleus Clusters

└ Pawlak (Dunajca MNCs) & Monet (Vetheuil MNC)

Pawlak (Dunajca MNCs) & Monet (Vetheuil MNC)

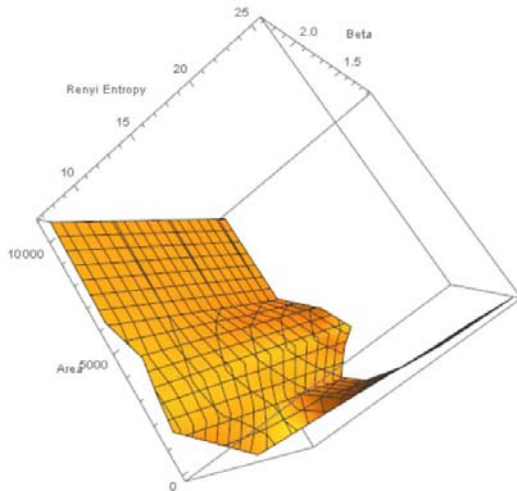
Pawlak Prezelom Dunajca MNCs



CM

# Pawlak (Dunajca 3D Entropy) & Monet (Vetheuil 3D Entropy)

## Pawlak Prezelom Dunajca 3D Entropy

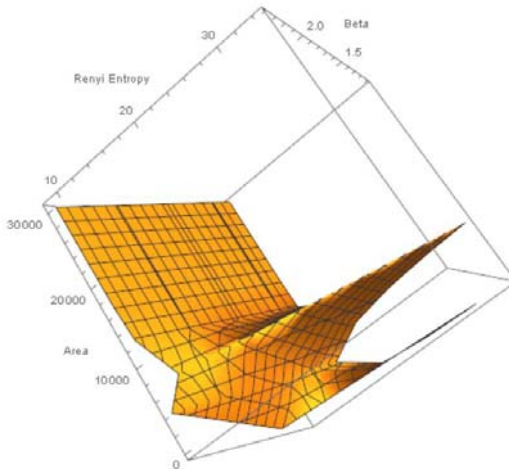
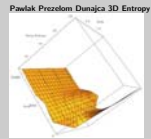


# Pawlak & Monet Painting Maximal Nucleus Clusters

2016-09-08

└ Pawlak (Dunajca 3D Entropy) & Monet (Vetheuil 3D Entropy)

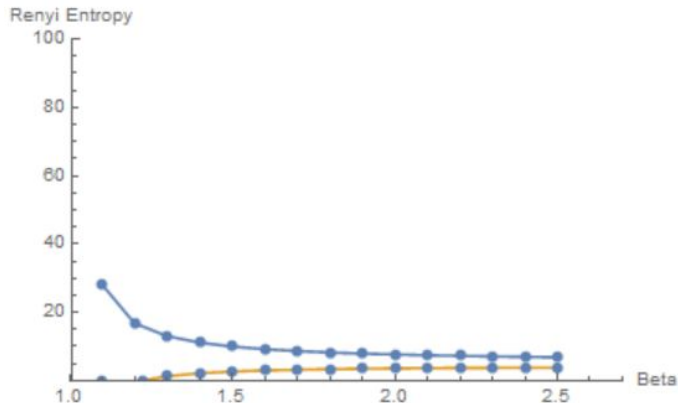
Pawlak (Dunajca 3D Entropy) & Monet (Vetheuil 3D Entropy)



CM

# Pawlak (Dunajca 2D Entropy) & Monet (Vetheuil 2D Entropy)

## Pawlak Prezelom Dunajca 2D Entropy



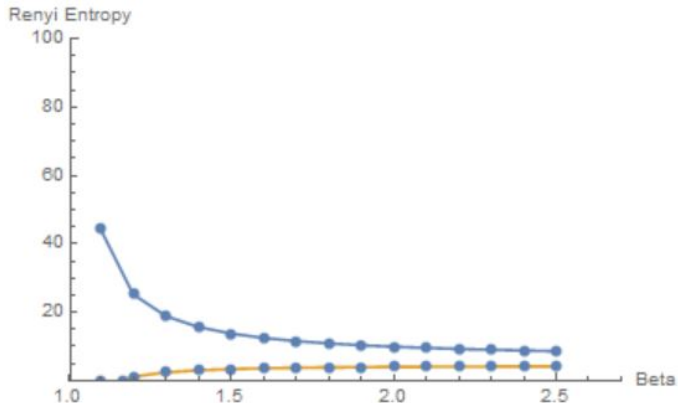
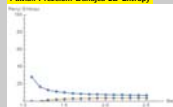
2016-09-08

# Pawlak & Monet Painting Maximal Nucleus Clusters

Pawlak (Dunajca 2D Entropy) & Monet (Vetheuil 2D Entropy)

└ Pawlak (Dunajca 2D Entropy) & Monet (Vetheuil 2D Entropy)

Pawlak Przełom Dunajca 2D Entropy



CM



## Pawlak Kampinoski Trail



# Pawlak & Monet Painting Maximal Nucleus Clusters

2016-09-08

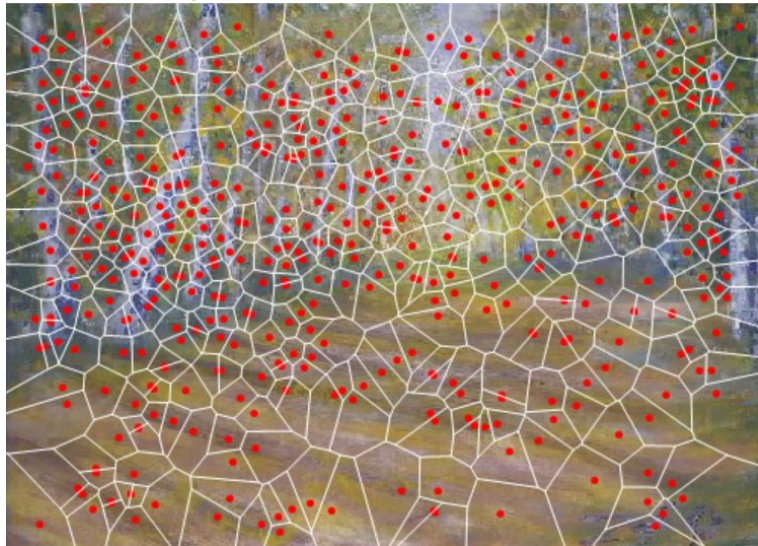
└ Pawlak (Kampinoski Park Naradowy) & Monet (Vetheuil)

Pawlak Kampinoski Trail



# Pawlak (Kampinoski Mesh) & Monet (Vetheuil Mesh)

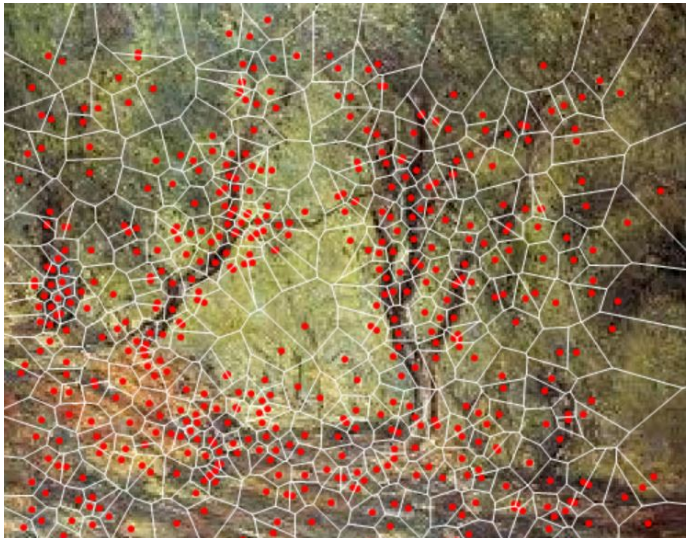
## Pawlak Kampinoski Trail Mesh



# Pawlak & Monet Painting Maximal Nucleus Clusters

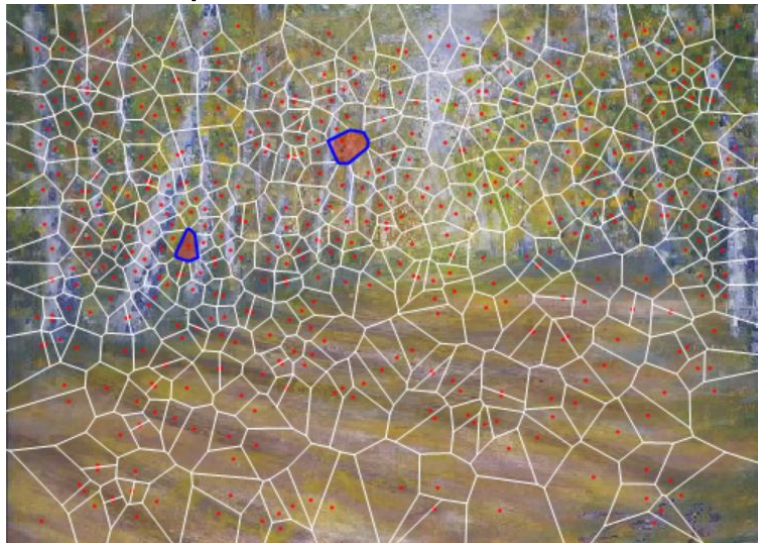
2016-09-08

└ Pawlak (Kampinoski Mesh) & Monet (Vetheuil Mesh)



# Pawlak (Kampinoski Nuclei) & Monet (Vetheuil Nucleus)

## Pawlak Kampinoski Trail Nuclei





# Pawlak & Monet Painting Maximal Nucleus Clusters

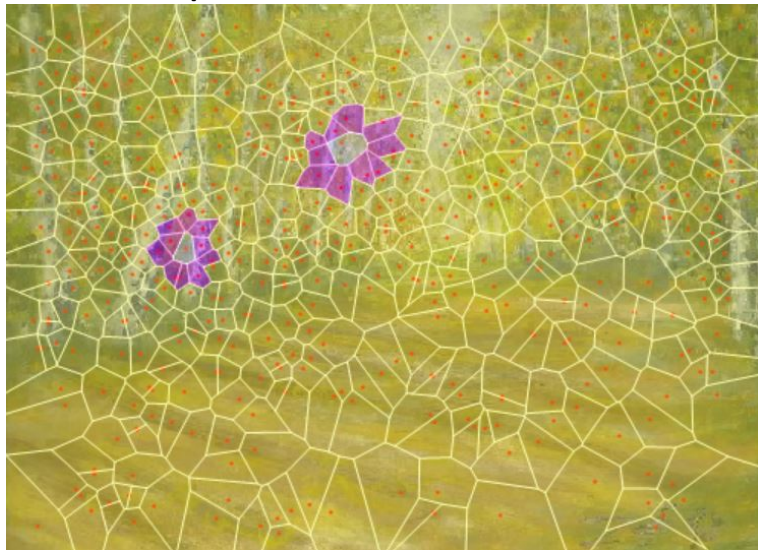
2016-09-08

└ Pawlak (Kampinoski Nuclei) & Monet (Vetheuil Nucleus)



# Pawlak (Kampinoski MNCs) & Monet (Vetheuil MNC)

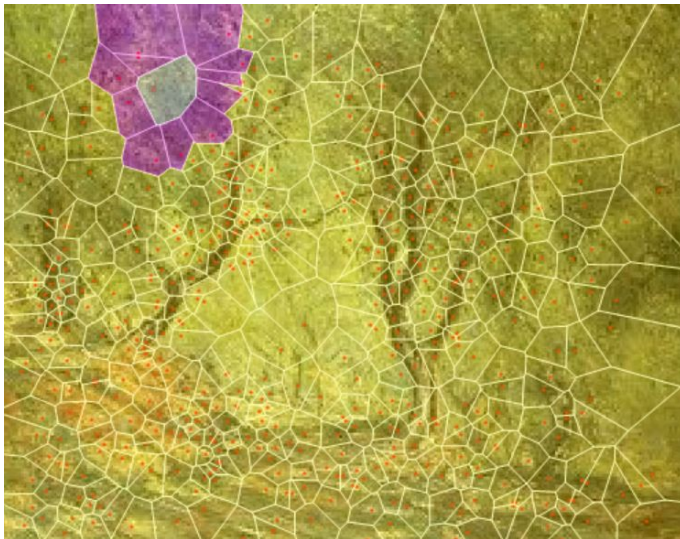
## Pawlak Kampinoski Trail MNCs



2016-09-08

# Pawlak & Monet Painting Maximal Nucleus Clusters

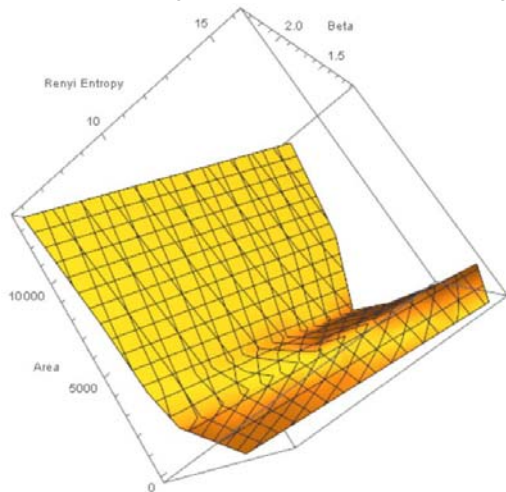
└ Pawlak (Kampinoski MNCs) & Monet (Vetheuil MNC)





# Pawlak (Kampinoski 3D Entropy) & Monet (Vetheuil 3D Entropy)

## Pawlak Kampinoski Trail 3D Entropy



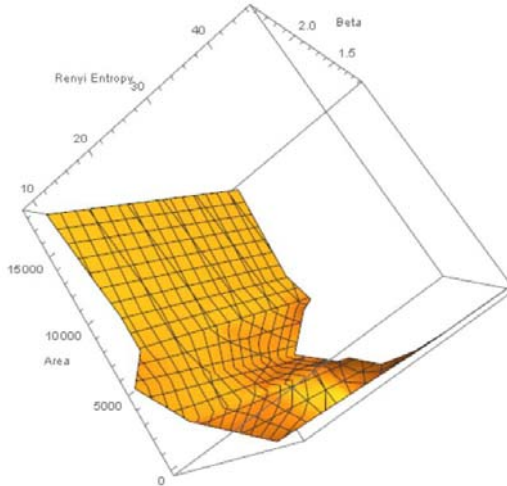
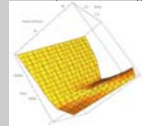
# Pawlak & Monet Painting Maximal Nucleus Clusters

2016-09-08

└ Pawlak (Kampinoski 3D Entropy) & Monet (Vetheuil 3D Entropy)

Pawlak (Kampinoski 3D Entropy) & Monet (Vetheuil 3D Entropy)

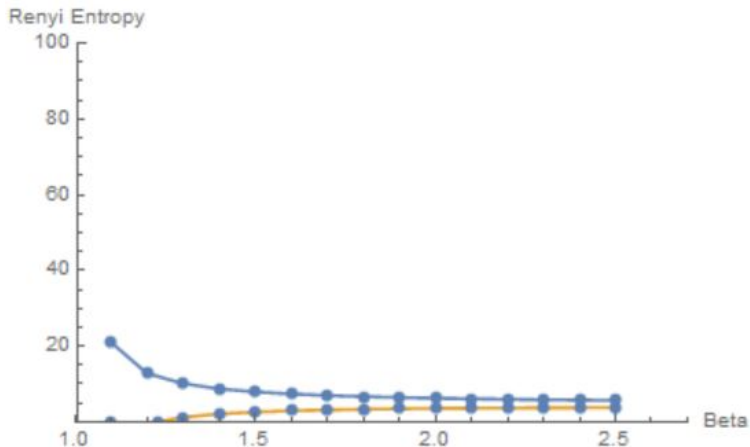
Pawlak Kampinoski Trail 3D Entropy



CM

# Pawlak (Kampinoski 2D Entropy) & Monet (Vetheuil 2D Entropy)

## Pawlak Kampinoski Trail 2D Entropy



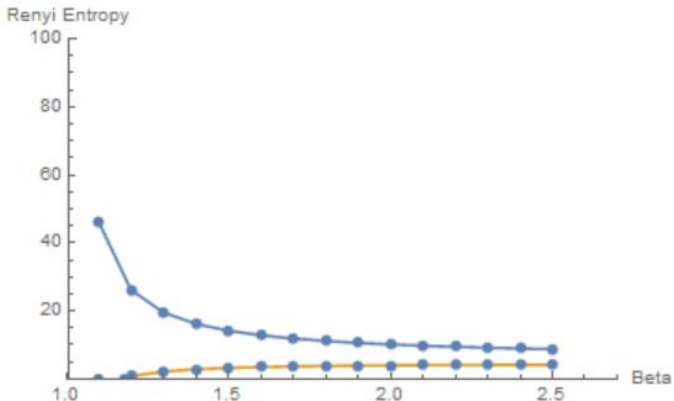
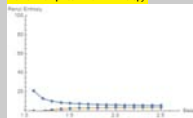
2016-09-08

## Pawlak & Monet Painting Maximal Nucleus Clusters

Pawlak (Kampinoski 2D Entropy) & Monet (Vetheuil 2D Entropy)

└ Pawlak (Kampinoski 2D Entropy) & Monet (Vetheuil 2D Entropy)

Pawlak Kampinoski Trail 2D Entropy



CM

# Pawlak (Dunajca Flowers) & Monet (Vetheuil Flowers)

## Pawlak Dunajca Flowers



2016-09-08

# Pawlak & Monet Painting Maximal Nucleus Clusters

└ Pawlak (Dunajca Flowers) & Monet (Vetheuil Flowers)

Pawlak Dunajca Flowers



CM

# Pawlak (Dunajca Flowers Mesh) & Monet (Vetheuil Flowers Mesh)

## Pawlak Dunajca Flowers Mesh



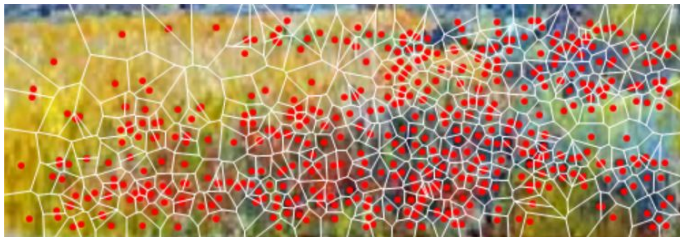
2016-09-08

## Pawlak & Monet Painting Maximal Nucleus Clusters

Pawlak (Dunajca Flowers Mesh) & Monet (Vetheuil Flowers Mesh)

└ Pawlak (Dunajca Flowers Mesh) & Monet (Vetheuil Flowers Mesh)

Pawlak Dunajca Flowers Mesh

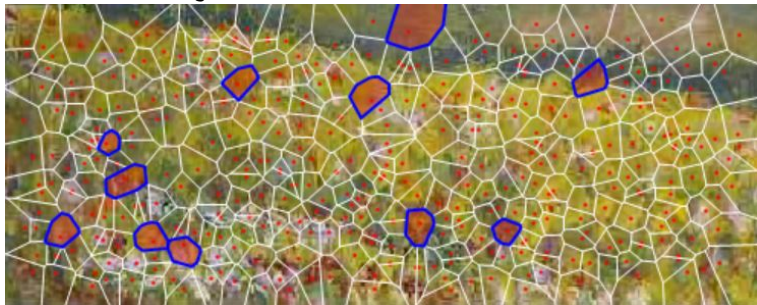


CM



# Pawlak (Dunajca Flowers Nuclei) & Monet (Vetheuil Flowers Nuclei)

## Pawlak Dunajca Flowers Nuclei



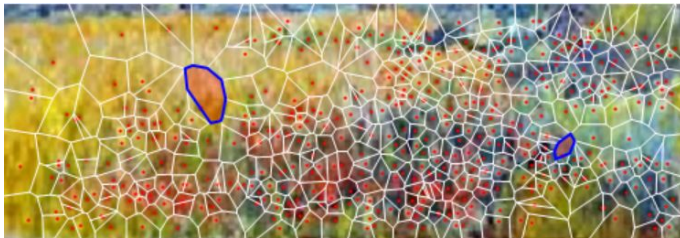
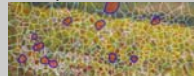
2016-09-08

## Pawlak & Monet Painting Maximal Nucleus Clusters

Pawlak (Dunajca Flowers Nuclei) & Monet (Vetheuil Flowers Nuclei)

└ Pawlak (Dunajca Flowers Nuclei) & Monet (Vetheuil Flowers Nuclei)

Pawlak Dunajca Flowers Nuclei



CM

# Pawlak (Dunajca Flowers MNCs) & Monet (Vetheuil Flowers MNCs)

## Pawlak Dunajca Flowers MNCs



2016-09-08

## Pawlak & Monet Painting Maximal Nucleus Clusters

Pawlak (Dunajca Flowers MNCs) & Monet (Vetheuil Flowers MNCs)

└ Pawlak (Dunajca Flowers MNCs) & Monet (Vetheuil Flowers MNCs)

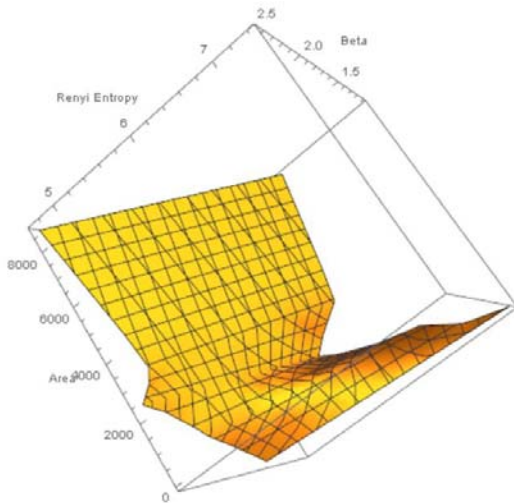
Pawlak Dunajca Flowers MNCs



CM

# Pawlak (Dunajca Flowers 3D Entropy) & Monet (Vetheuil Flowers 3D Entropy)

## Pawlak Dunajca Flowers 3D Entropy



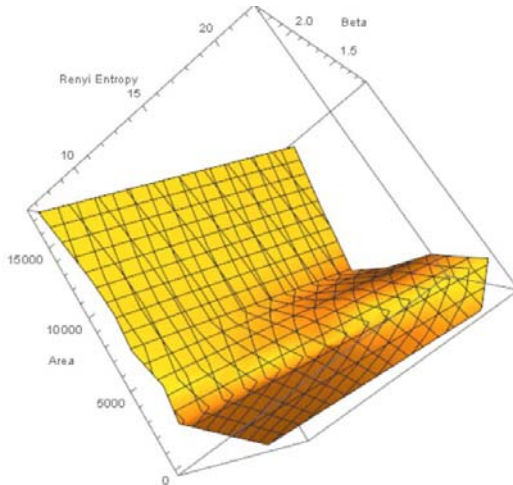
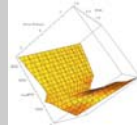
2016-09-08

## Pawlak & Monet Painting Maximal Nucleus Clusters

└ Pawlak (Dunajca Flowers 3D Entropy) & Monet (Vetheuil Flowers 3D Entropy)

Pawlak (Dunajca Flowers 3D Entropy) & Monet (Vetheuil Flowers 3D Entropy)

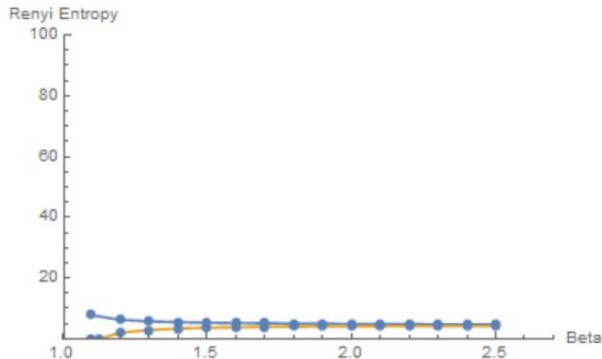
Pawlak Dunajca Flowers 3D Entropy



CM

# Pawlak (Dunajca Flowers 2D Entropy) & Monet (Vetheuil Flowers 2D Entropy)

## Pawlak Dunajca Flowers 2D Entropy



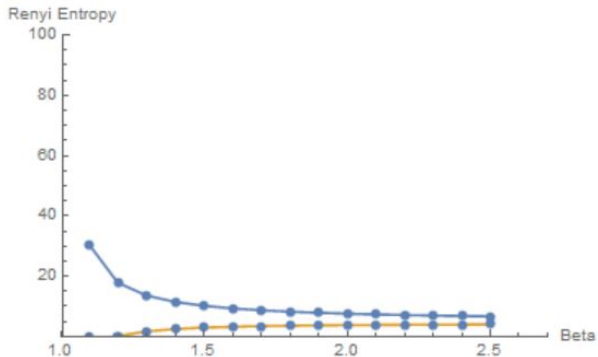
2016-09-08

## Pawlak & Monet Painting Maximal Nucleus Clusters

Pawlak (Dunajca Flowers 2D Entropy) & Monet (Vetheuil Flowers 2D Entropy)

└ Pawlak (Dunajca Flowers 2D Entropy) & Monet (Vetheuil Flowers 2D Entropy)

Pawlak Dunajca Flowers 2D Entropy



CM



## Landscapes and Nature Trails 1:

$$|\{\text{Nuclei}_{ZPawlak}\}| \geq |\{\text{Nuclei}_{CMonet}\}|.$$

## Landscapes and Nature Trails 2:

$$|\{\text{MNC}_{ZPawlak}\}| \geq |\{\text{MNC}_{CMonet}\}|.$$

## Wild Flowers:

$$|\{\text{MNCs}_{ZPawlak}\}| > |\{\text{MNCs}_{CMonet}\}|.$$

# References

- J.F. Peters: Computational Proximity. Excursions in the Topology of Digital Images, Intelligent Systems Reference Library, 102, Springer, Berlin, 2016.
- J. F. Peters and E. Inan: Strongly proximal edelsbrunner-harer nerves, Proceedings of the Jangjeon Mathematical Society, vol. 19, no. 3, pp. 563–582, 2016.
- J.F. Peters and A. Tozzi and S. Ramanna: Brain tissue tessellation shows absence of canonical microcircuits, Neuroscience Letters Journal, Elsevier, vol. 626, pp. 99-105, 2016.
- R. Hettiarachchi and J.F. Peters: Multi-Manifold-Based Skin Classifier on Feature Space Voronoi Regions for Skin Segmentation, Journal of Visual Communication and Image Representation, Elsevier *Accepted*.
- E. A-iyeh and J. F. Peters: Rényi entropy in measuring information levels in Voronoi tessellation cells with application in digital image analysis, Theory and Applications of Math. & Comp. Sci., vol. 6, no. 1, pp. 77–95, 2016
- R. Hettiarachchi and J.F. Peters: Multi-manifold LLE learning in Pattern Recognition, Pattern Recognition Journal, Elsevier, vol. 48, pp. 2947–2960, 2015.